A Distributed Heuristic for Energy-Efficient Multirobot Multiplace Rendezvous

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***Abstract*—We consider the problem of finding a set of trajectories for a heterogeneous group of mobile robots, such that a single service robot can rendezvous with every other member of the team in a prescribed order, and the total travel cost is minimized. We present a simple heuristic controller that runs independently on each robot, yet is guaranteed to achieve an approximation to global solution in bounded time linear in population size. Simulation experiments empirically show that the method finds approximated solutions of near-optimal quality on average.**

***Index Terms*—Mobile robot motion planning, multirobot systems.**

I. Problem Description and Characterization

CONSIDER a team of worker robots that can recharge by docking with a dedicated refueling (or equivalently, recharging) robot called a *tanker*, as described in [1]. The tanker robot could remain at a fixed location, acting as a conventional charging station, or it could move to rendezvous with worker robots. Simultaneously, worker robots can wait for the tanker to come to them, or they can move to meet the tanker.

As a generalization, the following natural problem can be stated: given a set of original locations of worker robots and tanker, find the set of meeting points such that the tanker meets every worker and minimizes the total energy spent on locomotion. Alternative objective functions include minimizing total rendezvous time, or a tradeoff between energy and time cost. Here, we address energy efficiency only. By analogy to a mother animal attending her offspring, we call this problem the "frugal feeding problem." If we impose a total order in which worker robots must be met and charged (perhaps based on urgency or some other priority scheme), we obtain the "ordered frugal feeding problem" considered in this paper.

We model locomotion costs as the weighted Euclidean distance between the origin and destination. The weight models the energetic cost of moving the robot some unit distance: a massive robot would have a higher weight than a small, lightweight robot.

The ordered frugal feeding problem can stated formally as follows.

Definition 1 (Ordered frugal feeding problem): Given original tanker location $r_0^0 = p_0 \in \mathbb{R}^d$, original worker locations $r_i^0 \in \mathbb{R}^d, i = 1, \ldots, k$, and locomotion cost functions $C_i : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}, i = 0, \ldots, k$, find

$$\min_{p_1, p_2, \ldots, p_k} \sum_{i=1}^{k} \left(C_0(p_{i-1}, p_i) + C_i(r_i^0, p_i) \right) \tag{1}$$

$$C_i(x, y) = w_i \|y - x\|. \tag{2}$$

Here, $C_0(x, y)$ gives the cost of tanker relocation from x to $y, C_i(x, y)$ gives the corresponding cost for worker i, and w_i is a weight of robot i.

Definition 1 could be amended to require the tanker to return to its original location after attending all workers, perhaps to refuel itself, without affecting the following analysis.

We denote solution points as p_i^*. The possibility of several robots being attended in one place is permitted, and captured by the possible coincidence of some meeting points.

The Fermat–Torricelli problem (also called the Steiner–Weber problem) asks for the unique point x minimizing the sum of distances to arbitrarily given points $x_1, \ldots, x_n$ in Euclidean d-dimensional space. Elaborating on the conventions in the Fermat–Torricelli problem literature [2], we identify some special cases of solution points as follows.

Definition 2 (Special cases for solution): If $p_i^* = r_j^0$ for some $j > 0$, we will call p_i^* *worker absorbed*. If $p_i^* = r_0^0$, we call p_i^* *tanker absorbed*. Otherwise, we call p_i^* *floating*.

If the meeting point for worker i is worker absorbed, worker i should either remain still and wait for the tanker to come to it, or it should move to the location of worker j ($i \neq j$). If a meeting point is tanker absorbed for worker i, the tanker should not move, and the worker should drive to the original tanker location. Floating meeting points do not coincide with any original robot location. Finally, there may exist hybrid solutions that combine absorbed and floating meeting points.

A. Analysis

The first observation to make is that solution points p_i^* cannot lie outside the convex hull of $\{r_0^0, r_1^0, r_2^0, \ldots, r_k^0\}$. This can be seen by considering a candidate meeting point outside the hull: replacing the candidate point with the closest point on the convex hull will unambiguously decrease the value of the goal function. Also, it is easy to prove the convexity of the objective function defined by (1) and (2) (see, e.g., [3, p. 239] for the idea of the proof).

Under certain conditions, we can quickly find solutions without solving the general problem. We first show the sufficient conditions for the worker absorbed case, i.e., where the meeting point for a worker is at worker's starting location.

Lemma 1: If $w_i \geq 2w_0$, then $p_i^* = r_i^0$. If $w_i > 2w_0$, then this solution is unique.

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Proof: The components of the objective function in (1), which depend on p_i, are

$$g_i(p_i){=}w_0\|p_i{-}p_{i-1}\| + w_0\|p_{i+1} - p_i\| + w_i\|p_i - r_i^0\|. \quad (3)$$

We can consider g_i in isolation and show that $g_i(p_i) \geq g_i(r_i^0)$ for $p_i \neq r_i^0$

$$\begin{aligned} g_i(p_i) &= w_0\|p_i - p_{i-1}\| + w_0\|p_{i+1} - p_i\| + w_i\|p_i - r_i^0\| \\ &\geq w_0\|p_i - p_{i-1}\| + w_0\|p_{i+1} - p_i\| + 2w_0\|p_i - r_i^0\| \\ &= w_0(\|p_i{-}p_{i-1}\|{+}\|p_i{-}r_i^0\|{+}\|p_{i+1}{-}p_i\| + \|p_i - r_i^0\|) \\ &\geq w_0(\|p_{i-1} - r_i^0\| + \|p_{i+1} - r_i^0\| = g_i(r_i^0). \end{aligned}$$

Here, the first inequality follows from the lemma statement and second inequality is a pair of triangle inequalities. Note that if the inequality in the lemma statement is strict, then the first inequality in above is also strict. In this case, r_i^0 will indeed be a unique optimal meeting point for robots 0 and i. ■

Now we show the sufficient conditions for the case where the tanker stands still and all workers are charged at the tanker location.

Lemma 2: If $\sum_{i=1}^k w_i \leq w_0$, then $p_i^* = r_0^0 = p_0 =$ for all i. If the inequality is strict, then this solution is unique.

Proof: Let $C_a = \sum_{i=1}^k w_i\|r_i^0 - p_0\|$ be the cost of a complete tanker absorbed solution, and $C(p_1, p_2, \ldots, p_k) = \sum_{i=1}^k (w_i\|r_i^0 - p_i\| + w_0\|p_i - p_{i-1}\|)$ denote the cost of an alternative solution. We will show that for all values of $p_i, i = 1, \ldots, k$, inequality $\Delta C = C_a - C(p_1, \ldots, p_k) \leq 0$ holds:

$$\begin{aligned} \Delta C = \sum_{i=1}^k & w_i(\|r_i^0 - p_0\| - \|r_i^0 - p_i\|) \\ & - w_0 \sum_{i=1}^k \|p_i - p_{i-1}\| \quad \text{(triangle inequality)} \\ \leq & \sum_{i=1}^k w_i\|p_i - p_0\| - w_0 \sum_{i=1}^k \|p_i - p_{i-1}\| \\ & (\text{let}\|p_j - p_0\| = \max_i\{\|p_i - p_0\|\})) \\ \leq & \sum_{i=1}^k w_i\|p_j - p_0\| - w_0 \sum_{i=1}^j \|p_i - p_{i-1}\| \\ & \text{(series of triangle inequalities)} \\ \leq & \sum_{i=1}^k w_i\|p_j - p_0\| - w_0\|p_j - p_0\| \leq 0. \end{aligned}$$

The case with strict inequality is argued similarly. ■

B. Related Work

The ordered frugal feeding problem can be addressed as a specific case of the continuous multiple facility location problem [4], where points $r_0^0, r_1^0, \ldots, r_n^0$ serve as the existing facilities, points $p_1, \ldots, p_n$ are new facilities, and the costs are set appropriately.

A variety of numerical methods exist for tackling this generalized problem. However, these numerical methods typically require centralized computation and do not scale well. The main contribution of this paper is introducing a simple and provably correct scalable decentralized heuristic that is observed to produce multipoint rendezvous with locomotion cost close to that of an optimal ordered frugal feeding solution in a number of realistic experimental settings.

The Weiszfeld algorithm [5] serves as a basis to many continuous multiple facility location problem algorithms (see [4] for a thorough review). Li [6] improves the slow convergence order of the Weiszfeld algorithm and identifies good convergence properties that hold in nondegenerate cases. Projected direction methods [7] offer an alternative to the Weiszfeld approach; however, they are arguably less suitable for large-scale computation [6]. Most of the parallel methods for facility location problems focus on discrete cases. However, Rosen and Xue [8] offer a parallel algorithm for continuous single facility location problem. The theoretical foundation for parallelization of gradient-like algorithms is given in [9].

Related robotics research includes studies of single-point rendezvous problems. Usually authors consider the setting where robots have incomplete information about locations of each other that makes it difficult to coordinate and agree on a single meeting point [10]–[13]. For example, Ando *et al.* [14] describe and prove the correctness of a distributed rendezvous algorithm for robots with limited visibility. Lanthier *et al.* [15] present an algorithm for finding the meeting point that minimizes the maximum individual travel costs to a single meeting point on a weighted terrain. Smith *et al.* [16] describe a scheme that makes the convergence process more organized in a certain mathematical sense.

In our previous work, we have described a distributed heuristic for a simple single-point rendezvous that minimizes total locomotion costs [17]. For the ordered frugal feeding problem, we have given two custom numerical algorithms that give approximate solutions, as well as a complete polytime discrete solution for the case when meeting points are restricted to a fixed set of locations and arbitrary locomotion cost functions in (1) [18]. The same paper also describes a general frugal feeding problem where the order of recharging is not fixed and is also subject to optimization. Reduction from the Euclidean traveling salesman problem (ETSP) to general frugal feeding problem shows that the finding optimal charging order is NP-hard [18]. This reduction does not imply that ETSP tour offers a good approximation to the solution to frugal feeding problem or even to the combinatorial part of the latter.

II. Distributed Heuristic for the Ordered Frugal Feeding Problem

A. Algorithm

We present a method whereby a simple controller, running in parallel on each robot, causes the robots to achieve low-cost multipoint rendezvous thus approximating the solution to the ordered frugal feeding problem. The method is scalable, in that both the total computation time and time to task completion are linear in the population size. Importantly, the per-robot, per-time-step cost is constant, so the method works for arbitrary population sizes.

This algorithm requires every worker to know its own weight and the tanker weight, whether or not it is the head of the current charging queue, and the direction toward the next worker in the queue and the previous worker (or tanker if the robot is at the head of the queue). The tanker needs to know its own weight, the weights of the first two robots in the queue and the directions toward them. As a robot is met and charged, it is removed from the queue, and this update is broadcast to all robots. All robots have first-order dynamic control. The algorithm is described and analyzed in discrete time but can be reformulated for continuous time.

We define a *meeting* as the event when robots come within distance s of each other. We denote the current location of robot i as r_i.

Algorithm 1 Distributed Heuristic

1: $i \leftarrow 1$
2: define $\vec{d}(x,y) = (y-x)/\|x-y\|$
3: **while** $i \le k$ **do**
4:   **if** $\|r_0 - r_i\| < s$ **then**
5:     tanker charges worker i; $i \leftarrow i+1$
6:     **next iteration**
7:   **if** $i = k$ (only one robot in queue) **then**
8:     the lighter of robots 0 and n goes towards the other
9:   **end**
10:   **for all** $j \in \{0, i, i+1, \ldots, k\}$ **do** {in parallel}
11: $$\vec{D}_j \leftarrow \begin{cases} w_i\vec{d}(r_0, r_i) + w_0\vec{d}(r_0, r_{i+1}), & \text{if } j = 0 \\ w_0(\vec{d}(r_i, r_0) + \vec{d}(r_i, r_{i+1})), & \text{if } j = 1 \\ w_0(\vec{d}(r_j, r_{j-1}) + \vec{d}(r_j, r_{j+1})), & \text{if } i < j < k \\ w_0\vec{d}(r_k, r_{k-1}), & \text{if } j = k \end{cases}$$
12:   **if** $\|\vec{D}_j\| < w_j$ **then**
13:     robot j stops
14:   **else**
15:     robot j proceeds in the direction $\vec{D}_j$

B. *Proof of Correctness and Run Time Bounds*

This approach is based on the embodied approximation philosophy described in [19]. Instead of operating with the model of the world and searching for the complete solution, each robot uses the position of itself, its queue predecessor and successor as the current approximation to the solution points. Every robot tries to improve the global solution quality by moving in the direction that decreases the part of the total cost function that concerns itself and its neighbors. If the robot finds itself located at the minimizing point for the current local configuration of robots, the robot stops. More formally, robots 0 (the tanker) and 1 (the next robot to be charged) move along the approximated antigradient of the cost function $g(p_1) = w_0\|r_0 - p_1\| + w_0\|p_1 - p_2\| + w_1\|p_1 - r_1\|$ using the current position of robot 2 as an approximation to the unknown solution point p_2. The rest of the robots $j, j = 2, \ldots, k$ move along the approximated antigradient of the cost functions $f_j(p_j) = w_0\|p_j - p_{j-1}\| + w_0\|p_j - p_{j+1}\| + w_j\|r_j - p_j\|$ using r_{j-1}, r_{j+1} as approximations for the unknown solution points p_{j-1}, p_{j+1}. Parallel computation of movement directions in step 11 and simultaneous movement of all robots in steps 12–15 provides the scalability of this algorithm. Unlike the single-point rendezvous heuristic described in [17], here the robot needs to know the direction to only two other robots to calculate its movement direction.

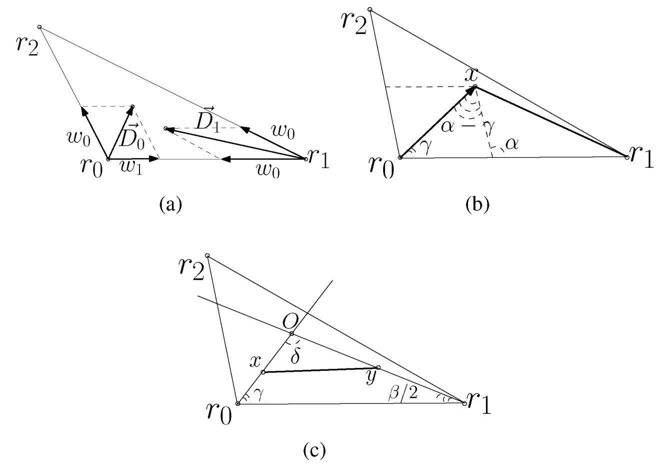

Fig. 1. Frugal feeding heuristic correctness proof. r_0 is the tanker robot, r_1 is the worker robot the tanker should meet next, r_2 is the worker robot to meet after r_1. (a) Robots and their direction vectors. (b) Tanker r_0 moves to point x, worker r_1 stops. (c) Tanker r_0 moves to point x, and worker r_1 moves to point y.

Later we prove that Algorithm 1 is correct and all of the workers are eventually charged. To do this, we bound the time required for the tanker to meet the first worker in the queue. Since the workers are removed from the queue after being charged, the same bound will apply consecutively to every worker becoming a head of the charging queue.

Theorem 1: For any initial locations $r_j^0, j = 0, \ldots, k$ and meeting range s if robots $0, 1$ recalculate their movement direction $\vec{D}_j$ every time they travel distance $\epsilon < s/2$, then after at most $\lceil 4\|r_0^0 - r_1^0\|^2/(\epsilon(s-2\epsilon))\rceil$ iterations, robots 0 and 1 will meet.

Proof: Consider the situation depicted in Fig. 1. We need to show that after robots 0 and 1 stop or move some distance ϵ along the directions $\vec{D}_0$ and $\vec{D}_1$ as prescribed by the algorithm, distance $\|r_0 - r_1\|$ decreases significantly enough that in a finite time this distance is smaller than meeting range s. The proof proceeds in four steps. First, we show that 0 and 1 will never satisfy their stopping conditions simultaneously before meeting, i.e., at least one of them moves. Second, we bound the decrease of the distance in one iteration in case when 0 stops and 1 moves. Then, we do the same for the case when 0 moves and 1 stops. Finally, we bound the decrease of distance for the case when both 0 and 1 move.

First, we establish the conditions for 0 and 1 to stop. $\angle r_2r_0r_1 = \alpha$ and $\angle r_2r_1r_0 = \beta$. Norms of direction vectors $\|\vec{D}_0\|^2 = w_1^2 + w_0^2 + 2w_1w_0\cos\alpha$ and $\|\vec{D}_1\|^2 = 2w_0^2(1+\cos\beta)$. Robot 0 stops when $\|\vec{D}_0\| < w_0$, or $w_1^2 + w_0^2 + 2w_1w_0\cos\alpha < w_0^2$, which, given positive weights, simplifies to

$$w_1 + 2w_0\cos\alpha < 0 \text{ (stopping conditions for } r_0). \tag{4}$$

Similarly, for robot 1, the stopping conditions are $\|\vec{D}_1\| < w_1$

$$2w_0^2(1+\cos\beta) < w_1^2 \text{ (stopping conditions for } r_1). \quad (5)$$

Note that since w_0 and w_1 are positive, (4) implies $\alpha > \pi/2$.

1) *Both robots stop:* Substituting $w_1^2 < 4w_0^2\cos^2\alpha$ from (4) into (5) and simplifying gives

$$(1+\cos\beta) < 2\cos^2\alpha. \quad (6)$$

As $\beta \le \pi - \alpha$, and $\alpha > \pi/2$, we have $\|\cos\beta\| \ge \|\cos\alpha\|$. Plugging this into (6) and simplifying gives

$$2\cos^2\beta - \cos\beta - 1 > 0. \quad (7)$$

Solving this for $\cos\beta$, we get $\cos\beta \in (-\infty, -1/2) \cup (1, \infty)$, which means $\beta > 5\pi/6$. Since $\alpha > \pi/2, \alpha + \beta > \pi$, which violates the triangle sum of angles property. Thus, the system of inequalities (4) and (5) has no valid solutions. This means robots 0 and 1 cannot stop simultaneously.

2) 0 *stops,* 1 *moves:* Initially, the distance between robots is $l_t = \|r_0 - r_1\|$. After 1 moves to point x traveling ϵ, the distance between robots becomes $l_{t+1} = \|r_0 - x\|$. Note that $\vec{D}_1$ always bisects $\angle\beta$.
Using law of cosines, $l_{t+1}^2 = l_t^2 + \epsilon^2 - 2l_t\epsilon\cos\beta/2$. Since $l_t > s$ and $\beta/2 < \pi/4$ (because $\alpha > \pi/2$), the following is valid:

$$\begin{aligned} l_{t+1}^2 &< l_t^2 + \epsilon^2 - 2l_t\epsilon\sqrt{2}/2 \\ &< l_t^2 + \epsilon^2 - \sqrt{2}l_t\epsilon < l_t^{\,2} + \epsilon^2 - \sqrt{2}\epsilon s. \end{aligned} \quad (8)$$

This allows to bound the decrease in the squared distance between robots in one iteration as

$$l_t^2 - l_{t+1}^2 > \epsilon(\sqrt{2}s - \epsilon). \quad (9)$$

3) 0 *moves,* 1 *stops:* This situation is illustrated by Fig. 1. Here, tanker 0 moves to point x, decreasing the distance to the worker from $l_t = \|r_0 - r_1\|$ to $l_{t+1} = \|r_1 - x\|$. The distance tanker travels is $\epsilon = \|x - r_0\|$. By the law of cosines

$$l_{t+1}^2 = l_t^2 + \epsilon^2 - 2\epsilon l_t\cos\gamma. \quad (10)$$

To bound γ, we apply the law of sines, $w_0/\sin\gamma = w_1/\sin(\alpha-\gamma)$. Starting with $w_0\sin(\alpha-\gamma) = w_1\sin\gamma$ and doing trigonometric transformations, we derive

$$\tan\gamma = \frac{w_0\sin\alpha}{w_1 + w_0\cos\alpha}. \quad (11)$$

Since $\beta \le \pi - \alpha, \cos\beta \ge \cos(\pi-\alpha)$. This inequality can be plugged into stopping condition (5) to get $2w_0^2(1 + \cos(\pi-\alpha)) < w_1^2$. The latter simplifies to $\cos\alpha > (2w_0^2 - w_1^2)/2w_0^2$. Plugging this into (11) and using $w_1 \le 2w_0$, we obtain

$$\begin{aligned} \tan\gamma &< \frac{w_0\sin\alpha}{w_1 + [(2w_0^2 - w_1^2)/2w_0]} \\ &< \frac{2w_0^2}{2w_0w_1 + 2w_0^2 - w_1^2} \le \frac{2w_0^2}{w_1^2 + 2w_0^2 - w_1^2} = 1. \end{aligned} \quad (12)$$

This implies $\gamma \in [0, \pi/4]$. We use this bound with (10) to arrive at the same bound on squared distance as (8) describes. Thus, the bound (9) applies for this case as well.

4) *Both robots move:* Fig. 1(c) shows robot 0 moving to point x and robot 1 moving to point y. The distance between robots changes from $l_t = \|r_0 - r_1\|$ to $l_{t+1} = \|x - y\|$. Both robots move equal distance $\|x - r_0\| = \|y - r_1\| = \epsilon$. We will use additional notation $a = \|O - r_0\|$ and $b = \|O - r_1\|$.
Using law of cosines

$$\begin{aligned} l_{t+1}^2 &= (a-\epsilon)^2 + (b-\epsilon)^2 - 2(a-\epsilon)(b-\epsilon)\cos\delta \\ &= l_t^2 + 2\epsilon(1-\cos\delta)(\epsilon - a - b). \end{aligned} \quad (13)$$

We start by bounding δ. Note that $\gamma + \beta/2 + \delta = \pi$. From (4) follows that 0 moves only when $w_1 + 2w_0\cos\alpha \ge 0$. Given positive weights, if $\cos\alpha \ge 0$, then $w_1 + w_0\cos\alpha > 0$. If $\cos\alpha < 0$, then $w_1 + w_0\cos\alpha = w_1 - w_0\|\cos\alpha\| > w_1 - 2w_0\|\cos\alpha\| = w_1 + 2w_0\cos\alpha \ge 0$ (again, given positive weights). Thus, we can use $w_1 + w_0\cos\alpha > 0$ in (11) to get $\tan\gamma > 0$. Thus, if r_0 moves, $\gamma \in [0, \pi/2]$. Now, consider two cases. First, if $\beta \ge \pi/2$, then $\gamma + \beta/2 \le \pi - \beta/2$, which implies $\delta \ge \pi/4$. Second, if $\beta < \pi/2, \gamma + \beta/2 < \pi/2 + \pi/4$. In this case, $\delta > \pi/4$. Thus, we bounded δ, and

$$1 - \cos\delta > 1 - \frac{\sqrt{2}}{2}. \quad (14)$$

Now, we need to bound $\epsilon - a - b$. Applying the law of sines, we get $l_t/\sin\delta = b/\sin\gamma = a/\sin(\beta/2)$. Thus

$$\begin{aligned} \epsilon - a - b &= \epsilon - \frac{l_t(\sin\gamma + \sin(\beta/2))}{\sin\delta} \\ &< \epsilon - \frac{s(\sin\gamma + \sin(\beta/2))}{\sin\delta} \\ &= \epsilon - s\frac{\sin\gamma + \sin(\beta/2)}{\sin(\gamma + \beta/2)} \\ &= \epsilon - s\frac{2\sin[(2\gamma+\beta)/4]\cos[(2\gamma-\beta)/4]}{2\sin[(2\gamma+\beta)/4]\cos[(2\gamma+\beta)/4]} \\ &= \epsilon - s\frac{\cos[(2\gamma-\beta)/4]}{\cos[(2\gamma+\beta)/4]} \\ &\le \epsilon - s\cos\left(\frac{2\gamma-\beta}{4}\right) \\ &= \epsilon - s(\cos(\gamma/2)\cos(\beta/4) + \sin(\gamma/2)\sin(\beta/4)) \\ &< \epsilon - s\cos(\gamma/2)\cos(\beta/4) \\ &< \epsilon - (\sqrt{2}/2)^2 s = \epsilon - \frac{s}{2}. \end{aligned} \quad (15)$$

Combining (13)–(15), we bound the decrease in squared distance

$$l_t^2 - l_{t+1}^2 > (2 - \sqrt{2})\epsilon\left(\frac{s}{2} - \epsilon\right) > \frac{\epsilon}{2}\left(\frac{s}{2} - \epsilon\right). \quad (16)$$

TABLE I
STATISTICS OF APPROXIMATION FACTOR BOUNDS

	$w_i = 1$	$w_i \sim U[1,3)$	$w_i \sim U[1,100]$
Mean	1.19	1.22	1.31
Median	1.19	1.20	1.25
Standard deviation	0.03	0.08	0.23
Skewness	0.67	1.18	4.27
Kurtosis	0.65	1.65	31.82

Comparing (9) and (16), we see that the latter is more conservative estimate of the squared distance decrease. Thus, if the initial distance between robots is L, the upper bound on the number of steps needed to meet is $\lceil 4L^2/(\epsilon(s-2\epsilon))\rceil$. ■

Corollary 1: Consider any initial locations $r_j^0, j = 0, \ldots, k$, meeting distance s and assume that charging occurs simultaneously. If robots recalculate their movement direction $\vec{D}_j$ every time they travel distance $\epsilon < s/2$, then after at most $\lceil k(4\max_{i,j}\|r_i^0 - r_j^0\|^2)/\epsilon(s-2\epsilon)\rceil$ iterations, all workers will be charged.

III. Experiments

Having shown that the method succeeds in bounded time, we seek to evaluate the quality of the solutions it generates. This is complicated because 1) the continuous problem has no known closed-form solution, and 2) no quality bounds are available for numerical approximation approaches. However, we can obtain optimal solutions for the discrete version of the problem [18], and we can provide an upper bound on the cost introduced by the discretization. So our empirical evaluation procedure is as follows: 1) Generate a set of robot weights and start positions within a bounding rectangle. 2) Find the cost of the optimal solution to the corresponding discretized problem, where robot start positions are quantized to a regular grid within the bounding rectangle. A lower bound on the cost of the optimal continuous solution is given by subtracting from this the maximum possible additional cost due to discretization. 3) Find the upper bound of approximation factor by dividing the frugal feeding heuristic cost by the lower bound of optimal cost.

Applying this methodology, we performed 3000 simulations running the frugal feeding heuristic on randomly generated instances of the problem, with initial robot locations drawn from the same uniform distribution, and three different uniform distributions of weights, with 1000 experiments for each weight distribution. In each experiment, ten randomly weighted worker robots and a tanker were placed at random locations in an square arena with 20 m sides. A meeting range of 0.1 m and a movement step length of 0.01 m were set. The path traveled by each robot was recorded, and its length was multiplied by the robot's weight, then summed for the population to give the total energy spent on performing the rendezvous. The arena was discretized using a 40×40 grid, giving a resolution of 0.5×0.5 m.

Table I reports the statistics of these approximation factor bounds for each distribution. For example, in the setting where all robots have weight of 1, we observed that the frugal feeding heuristic solution cost is on average no worse than 1.19 times the best possible solution.

The best approximation is achieved when all robots have identical weights. The results show increasing variation of the approximation factor bounds with increasing variation in robot weights. This is due to the method's use of only local information about neighboring robots in the rendezvous queue. To see this, consider a queue of three workers to be met, with weights $[1, 1, 10]$. The tanker will not consider the third worker until it has met the first, so the tanker may move suboptimally away from the high-weight robot at first. The approximation factor can be made arbitrarily bad by increasing the weight of the tanker and the third worker.

As the support of the weight distribution expands, the probability of this effect increases. Hence, the tail of the approximation factor distribution becomes more fat and the kurtosis increases. However, most of the probability remains concentrated around good approximations; hence, the skewness of the distribution also grows and the mean does not increase much.

If we know in advance that motion weights are likely to vary widely, this effect can be reduced (but not eliminated) by increasing the number of neighboring robots considered by the tanker and/or workers, with a corresponding constant factor increase in computation, sensing, and communication cost.

Summarizing, the average quality of approximated solutions for uniformly distributed problem appears very good. Therefore, the method could be used where the guaranteed quality of results obtained in a nonscalable centralized manner should be sacrificed in favor of a simple decentralized scalable solution with good average performance.

IV. Conclusion

In this paper, we considered the natural optimization problem of finding an energy-efficient simultaneous motion plan for a heterogeneous robot system where a single service robot needs to meet a number of worker robots in a specified order: the ordered frugal feeding problem. We presented the frugal feeding heuristic that exploits embodiment and the natural parallelism of the robot system to achieve high-quality approximated solutions in arbitrary population sizes with trivial computation per-robot per-time step and using information that should be feasible to obtain in practice. We established the correctness of the method by bounding convergence time and gathered empirical evidence of its average solution quality by experiment.

This paper can be extended in several directions. The definition of the problem can be amended to include multiple tankers, adding an interesting coordination challenge. Also of interest are more complex cost functions, particularly those involving time constraints, joint time-and-energy constraints, and the dynamics of the robots. A challenging extension is to find good solutions to the general frugal feeding problem, including the order of robot meetings. Finally, it would be useful to evaluate the frugal feeding heuristic in dynamic and uncertain environments.

References

[1] P. Zebrowski and R. Vaughan, "Recharging robot teams: A tanker approach," in *Proc. Int. Conf. Adv. Robot. (ICAR)*, Seattle, WA, Jul. 2005, pp. 803–810.

[2] Y. Kupitz and H. Martini, “Geometric aspects of the generalized Fermat–Torricelli problem,” *Bolyai Soc. Math. Stud.*, vol. 6, pp. 55–129, 1997.

[3] H. M. V. Boltyanski and V. Soltan, *Geometric Methods and Optimization Problems*. Norwell, MA: Kluwer, 1999.

[4] Z. Drezner, Ed., *Facility Location. A Survey of Application and Methods*. New York: Springer-Verlag, 1995.

[5] E. Weiszfeld, “Sur le point pour lequel la somme des distances de n points donnes est minimum,” *Tohoku Math. J.*, vol. 43, pp. 355–386, 1937.

[6] Y. Li, “A Newton acceleration of the Weiszfeld algorithm for minimizing the sum of euclidean distances,” *Comput. Optim. Appl.*, vol. 10, no. 3, pp. 219–242, 1998.

[7] P. H. Calamai and A. R. Conn. (1980). A stable algorithm for solving the multifacility location problem involving euclidean distances. *SIAM J. Sci. Stat. Comput.* [Online]. vol. 1, no. 4, pp. 512–526. Available: http://link.aip.org/link/?SCE/1/512/1.

[8] J. Rosen and G.-L. Xue, “Computational comparison of two algorithms for the euclidean single facility location problem,” *ORSA J. Comput.*, vol. 3, no. 3, pp. 207–212, 1991.

[9] J. N. Tsitsiklis, D. P. Bertsekas, and M. Athans, “Distributed asynchronous deterministic and stochastic gradient optimization algorithms,” *IEEE Trans. Autom. Control*, vol. AC-31, no. 9, pp. 803–812, Sep. 1986.

[10] G. Dudek and N. Roy, “Multi-robot rendezvous in unknown environments, or, what to do when you’re lost at the zoo,” in *Proc. AAAI Nat. Conf. Workshop Online Search*, Providence, RI, Jul.1997, pp. 22–29.

[11] J. Cortes, S. Martinez, and F. Bullo, “Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions,” *IEEE Trans. Autom. Control*, vol. 51, no. 8, pp. 1289–1298, Aug. 2006.

[12] A. M. J. Lin and B. D. O. Anderson, “The multi-agent rendezvous problem,” in *Proc. 42th IEEE Conf. Decis. Control*, Maui, HI, 2003, pp. 1508–1513.

[13] K. Schlude, “From robotics to facility location: Contraction functions, Weber point, convex core,” Comput. Sci., Swiss Federal Inst. Technol. (ETHZ), Zurich, Switzerland, Tech. Rep. 403, 2003.

[14] H. Ando, Y. Oasa, I. Suzuki, and M. Yamashita, “A distributed memoryless point convergence algorithm for mobile robots with limited visibility,” *IEEE Trans. Robot. Autom.*, vol. 15, no. 5, pp. 818–828, Oct. 1999.

[15] M. A. Lanthier, D. Nussbaum, and T.-J. Wang, “Calculating the meeting point of scattered robots on weighted terrain surfaces,” in *Proc. 2005 Australasian Symp. Theory Comput. (CATS 2005)*. Darlinghurst, Australia: Australian Comput. Soc., 2005, pp. 107–118.

[16] S. Smith, M. Broucke, and B. Francis, “Curve shortening and the rendezvous problem for mobile autonomous robots,” *IEEE Trans. Autom. Control*, vol. 52, no. 6, pp. 1154–1159, Jun. 2007.

[17] P. Zebrowski, Y. Litus, and R. T. Vaughan, “Energy efficient robot rendezvous,” in *Proc. 4th Can. Conf. Comput. Robot Vis.*, May 2007, pp. 139–148.

[18] Y. Litus, R. T. Vaughan, and P. Zebrowski, “The frugal feeding problem: Energy-efficient, multi-robot, multi-place rendezvous,” in *Proc. IEEE Int. Conf. Robot. Autom.*, Apr. 2007, pp. 27–32.

[19] Y. Litus and R. Vaughan, “Distributed gradient optimization with embodied approximation,” in *Artificial Life XI: Proc. Eleventh Int. Conf. Simulation Synthesis Living Syst.*, S. Bullock, J. Noble, R. Watson, and M. A. Bedau, Eds. Cambridge, MA: MIT Press, 2008, pp. 359–365.

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